Determination of Energy and Electric Capacity of On-Board Supercapacitor Regenerative Energy Storage

Purpose. Development of a method for determining the main functional parameters of on-board supercapacitor recuperative energy storage based on the asymptotic theory of extreme statistics by Gumbel, taking into account stochastic nature of changes in recuperated voltage and current. Methodology. To achieve this purpose, methods, devices and computer systems for temporary registration of recuperated voltages and currents on operating electric locomotives, methods of the theory of random processes and methods of probabilistic and statistical processing of registrograms of voltages and currents were used. Findings. A computational and experimental method for estimating recuperative energy has been proposed and practically applied. A probabilistic method has been developed for determining the energy and electric capacity of on-board supercapacitor recuperative energy storage units. Numerical probabilistic and statistical calculations of the energy and electric capacity of on-board storage for the VL8 and VL11M6 electric locomotives during their operation in the sections of Prydniprovska railway have been carried out. It was found that the energy and electric capacity of on-board storage devices are distributed according to an exponential law with a clear prevalence of their minimum values and in compliance with direct proportionality between them. Originality. For the first time, an autonomous phase of recuperative braking mode of an electric rolling stock has been developed, which makes it possible to significantly reduce the mass and dimension of a supercapacitor storage. The asymptotic theory of extreme statistics by Gumbel was adapted to the method for calculating energy and electric capacity of an on-board storage device, which made it possible to take into account the influence of stochastic nature of changes in the recuperated voltage and current. The probabilistic influence regularities of the change nature in the recuperation energy on the capacity of on-board storage in the phase of recuperative braking have been established. Further development was obtained by a computational-experimental method for assessing the recuperative energy, based on monitoring and using the time dependences of voltage and current obtained in real modes of recuperative braking. For the first time in electric traction systems, it was proposed to carry out the transition from the recuperative braking mode to the recuperative regeneration mode. Practical value. The developed method and technique based on it make it possible to evaluate functional parameters of on-board storage device of all types of electric rolling stock, considering stochastic nature of recuperated voltages and currents. Numerical-graphic dependences of the energy intensity and capacity of the on-board storage are recommended for predicting and evaluating these parameters for...
various modes of recuperative braking. Since the task of designing an on-board storage unit (in terms of mass and dimensions) is ambiguous, therefore, in each specific case of the type of electric locomotive and recuperation modes, it must be solved individually, taking into account the probability of the corresponding capacitance values.

**Keywords:** on-board storage; energy capacity; supercapacitor; recuperative energy; random process; voltage; current; extreme statistics

**Introduction**

The main disadvantages of the existing system of regenerative braking of electric rolling stock (ERS) of direct current, preventing its efficient implementation, are caused by the necessary connection, that is, non-autonomy of recuperating ERS with the traction power supply system [5, 14]. Therefore, currently, the issue of increasing the reliability, stability and electric power efficiency of recuperative braking (RB) is particularly acute. In our opinion, this problem can be solved by transition from the systems and modes of RB to the regenerative braking systems [5].

The term «regeneration» itself (Latin regeneratio – regeneration) denotes the restoration of the initial qualities of the waste product, i.e. the renewal of electric energy of electrodynamic braking. In the definitions of the concept of regeneration, there is no condition for the return of regenerated electricity to the supply network. This necessitates the need for a regeneration energy storage at the ERS itself – an onboard storage unit operating in a buffer mode. The on-board buffer energy storage unit (SU) means device, whose primary function is the operational use of electrical braking energy by accumulation, short-term storage and its subsequent implementation in the traction mode.

Currently available domestic and foreign publications show that the most energy-efficient and, therefore the most prospect, are the on-board (on the ERS) SU systems based on supercapacitors generally called «electrical double layer capacitors» (EDLC) [5]. However, at the same time, SU must have a number of certain parameters.

The basic parameters determining the functional capabilities of the use of energy storage units in energy systems are as follows: maximum active power $P_{\text{max}}$; maximum energy capacity $E_{\text{max}}$; operation time $t_{\text{op}}$; energy reversal time $t_{\text{rev}}$. The energy storage unit is located stationary in the electric power system itself. Such parameters are also inherent in stationary SU, used in the electric transport systems provided that these parameters are designed to receive all the energy recovered for a trip in an electric rolling stock [8]. In this case, the mass and dimension will be insignificant. An on-board storage device, including a supercapacitor, should perform a completely different function. To reduce the storage unit capacity $C$, it should operate in the so-called phase regeneration mode: receive the electric energy, recovered in the specific phase of recuperative braking and then, in the next movement phase, promptly give it to a specific consumer of the ERS. Consequently, all the SU parameters must be designed for the one phase RB energy. This mode is advisable to reduce the capacity $C$, and, consequently, the mass and dimensions of the SU, as well as the energy reversal time. Thus, one of the main functional parameters of the on-board SU should be taken as follows: maximal power and maximal energy capacity, as well as the minimum ones: capacity, weight and dimensions, and recuperation energy reversal time.

The SU power should be equal to the ERS recuperation power in one phase of the RB, and its energy capacity – to the recuperation energy $W_p$ also for one phase of the RB. The value of $W_p$ is basic for determining the main SU parameter – its capacity, as well as for assessing the functional capability of using this $W_p$.

The energy that is accumulated in the electric field of the capacitor in the RB phase is determined by the expression:

$$W_p = \frac{C U_r^2}{2}, \quad (1)$$

where $U_r$ – voltage at which the recuperation energy was obtained in the RB phase. According to (1) the required storage capacity is:

$$C = \frac{2W_p}{U_r^2}. \quad (2)$$
In (1) and (2), the values $W_r$ and $U_r$ are random. Therefore, in practical calculations of $C$, it is necessary to solve the problem, which values of $W_r$ and $U_r$ (maximum, minimum or average) should be used. From the point of view of the sufficiency of the value $C$ to receive the entire $W_r$, it is necessary to take the maximum values of $W_r$, but in this case, an excessive volume of storage is not excluded. In this regard, for some optimization of this approach, we will apply the Gumbel asymptotic theory of extreme statistics, i.e., the distribution laws and their probabilistic characteristics of the extreme (maximum and minimum) values of the sample of the random variable $X$ under study.

For the first time, a relatively detailed methodology for calculating condenser on-board recuperation energy storage units was described in work [6].

The methodology for determining the rational parameters of supercapacitor recuperation energy storage unit in the urban transport system is presented in [3]. The author proposes only mass and cost of on-board storage as the criteria for assessing the rational parameters.

As follows from the above-mentioned and a number of other sources, they mainly discuss two issues: the first is possible storage location options; the second—its cost indicators. Secondly, the issues of on-board supercapacitor storage units are considered only for urban transport systems (trams, trolleybuses, underground motor coaches), which is caused by the possibility of calculating the energy intensity and capacity of the storage unit for the entire trip. For mainline transport systems (in particular, for electric locomotives), this is impractical based on the large mass and dimensions of the storage, which explains the lack of publications on these systems. To solve this problem, another approach is needed. And, finally, thirdly, all authors assess the required energy and electrical capacities of supercapacitor storage units for deterministic (non-random) recuperable voltages and currents, while they are stochastic, often change abruptly (Fig. 1). This is what determines the assessment of the rational parameters of the capacitive storage with a large error.
The article provides for the development of a method for determining the main functional parameters of on-board supercapacitor recuperative energy storage in its phase regeneration mode of operation and taking into account the stochastic (random) nature of changes in the processes of recovered voltage and current based on the asymptotic theory of extreme statistics by Gumbel.

Methodology

Probabilistic and statistical calculations of the energy intensity and capacity of on-board storage units were carried out based on the registrograms (time dependences) of voltage on the current collector \( U(t) \) and the recovered current \( I(t) \), were obtained in real operating conditions of the VL8 and VL11M6 electric locomotives in the Prydniprovska railway. 30 synchronously recorded voltage and current realizations were obtained and processed according to the method in [15]. Receiving registrogram sections, where the current had negative values, were considered to be recuperation current.

The recuperation energy \( W_p \) was determined by the experimental-calculation method [5], based on the time dependences of the recuperable voltages and currents. This method was positively different from the existing ones in the fact that the calculations were performed based on modern concepts and formulas of powers and energy in electrical circuits with non-sinusoidal electrical values, while the existing measuring instruments and systems were based on the concepts and techniques adopted back in the 40s of the twentieth century.

Based on this method, according to [5], the recuperation energy is determined as follows (Fig. 2):

\[
W_p = \frac{1}{N} \sum_{n=1}^{N} u_n i_n \Delta t ,
\]

where \( u_n, i_n \) – instantaneous voltage and current values; \( N \) – total number of quantization points during 0-T time.

Theoretical aspects of the determining storage capacity. Obtaining the distribution laws of extreme values is possible in two ways. The first involves performing experimental and statistical tests of the value of \( X_v \) in strictly identical conditions for all experiments, which is practically unfeasible for the recuperation phases.

The second way is analytical, based on the use of the distribution function \( F(x) \) of the investigated random variable \( X \), constructed earlier for the entire sample size, that is, for all \( n \) values (the entire population) of \( X \).

Let us assume that \( n \) measurements are carried out for a random variable \( X \) having a distribution function \( F(x) \) and a probability density \( f(x) \). The
measurement results give a sequence of numbers 
\(x_{n1} \leq x_{n2} \leq \cdots \leq x_{nn}\). One should find the distribution function \(F_{nn}(x)\) and the probability density \(f_{nn}(x)\):

\[
f_{nn}(x) = \frac{dF_{nn}(x)}{dx}, \quad (4)
\]

for the maximum values of the random variable \(x_{nn}\) in a set of \(n\) measurements. The distribution function \(F_{nn}(x)\) is nothing but the probability of finding the inequality \(X < x\) in each of \(n\) measurements. If we take the scheme of independent tests, then the problem is extremely simple to solve. The probability of finding the inequality \(X < x\) as a result of one measurement is, obviously, \(F(x)\). Hence, according to the probability multiplication theorem:

\[
F_{nn}(x) = P_n(X < x) = F^n(x).
\]

Application of formula (4) gives:

\[
f_{nn}(x) = nF^{n-1}(x) p(x). \quad (5)
\]

A similar problem can be set for the distribution of the minimum values of \(x_{n1}\). The corresponding characteristics will be denoted by \(F_{n1}(x)\) and \(p_{n1}(x)\). According to the definition, the function \(1 - F_{n1}(x)\) is equal to the probability of finding the inequality \(X > x\) as a result of each of \(n\) measurements. Noting that according to the probability multiplication theorem

\[
P_{n}(X > x) = \left[P_{n}(X > x)\right]^{n} = \left[1 - F(x)\right]^{n},
\]

we will find

\[
F_{n1}(x) = 1 - \left[1 - F(x)\right]^{n}.
\]

Differentiating the distribution function, we will obtain:

\[
f_{n1}(x) = n\left[1 - F(x)\right]^{n-1} p(x). \quad (6)
\]

The distribution laws \(F_{nn}(x)\), \(f_{nn}(x)\), \(F_{n1}(x)\), \(f_{n1}(x)\) and their parameters of extreme values are determined by the nature of the investigated random variable, size of \(n\) sample and the initial function \(F(x)\) of its distribution. In this case, with an increase in \(n\), the most probable values of the maximums of the random variable \(X\) shift to the right, and the minimum ones – to the left.

If \(n\) is large, then using expressions is hampered. Therefore, in the works [2, 3], the asymptotic properties of extreme values distributions with \(n \to \infty\) were investigated and three so-called limit distributions Gumbel were obtained – the first, the second, and the third. Each of them is applicable under certain restrictive conditions with respect to the investigated \(X\). In particular, if the quantity \(X\) is limited both from below and from above in the interval of its existence \([a, b]\) (which is typical for the values \(W_p\) and \(U_p\), then it obeys the third limiting distribution by Gumbel [3]:

for maximal values:

\[
F_{nn}(x) = \begin{cases} 
\exp \left[ -\left( \frac{b-x}{\nu} \right)^{\beta} \right] & \text{при } x < b, \\
1 & \text{при } x \geq b;
\end{cases} \quad (7)
\]

for minimal values:

\[
F_{n1}(x) = \begin{cases} 
1 - \exp \left[ -\left( \frac{x-a}{\nu} \right)^{\beta} \right] & \text{при } x > a, \\
0 & \text{при } x \leq a,
\end{cases} \quad (8)
\]

where \(\beta\) and \(\nu\) – some positive numbers.

For not too large \(n\) (which is typical for \(W_p\) and \(U_p\)) and the initial Gaussian distribution, the distribution density of the maximum values of the investigated random variable \(X\) can be determined by the formula:

\[
f_{nn}(x) = n \left[ 0.5 + \Phi \left( \frac{x-a}{\sigma} \right) \right]^{n-1} \cdot \frac{1}{\sqrt{2\pi}\sigma} \times \exp \left[ -\left( \frac{x-a}{3\sigma^2} \right)^2 \right]; \quad (9)
\]

for the minimal values:

\[
f_{n1}(x) = n \left[ 0.5 - \Phi \left( \frac{x-a}{\sigma} \right) \right]^{n-1} \cdot \frac{1}{\sqrt{2\pi}\sigma} \times \exp \left[ -\left( \frac{x-a}{2\sigma^2} \right)^2 \right]. \quad (10)
\]
where \( F \left( \frac{x-a}{\sigma} \right) \) – is known Laplace function; 
\( a, \sigma \) – respectively, the mathematical expectation 
and the mean-square deviation of the total sample, 
that is, the function \( F(x) \).

In this case, the mathematical expectations and 
mean-square deviations of the extreme values of the 
investigated random variable \( X \) are determined by 
the formulas:

\[
\overline{X} = M[X] = a \pm \sigma \cdot \sqrt{\ln n};
\]

\[
\sigma_{nm} = \sigma_{n1} = \frac{\pi \sigma}{\sqrt{6 \ln n}}.
\]

In expression (11), the plus sign refers to the 
maximum values, that is, \( \overline{X}_{nm} \) is determined, and the 
minus sign refers to the minimum values.

### Findings

The energy \( E_n \) and electric \( C \) capacities of the 
on-board storage units of recuperated electricity for 
the VL11M6 and VL8 electric locomotives oper-
ad with freight trains on the Nyzhnodniprovsk-
Junction – Chaplino section of the Prydniprovska 
railway were calculated. In this case, the energy ca-
pacity is taken to be equal to the recuperation energy 
\( W_p \) and is determined by the above experimental-
calculation method based on the voltage \( U_p \) and cur-
rent \( I_p \) obtained (and processed according to [4, 15]) 
under real conditions of the recuperative braking 
mode. The capacity was assessed for three cases: ac-
cording to the average values of \( W_p, U_p \) for the RB 
phase, according to formula (2) as the most probable 
value of the statistical values, according to the sta-
tistics of the maximum values by Gumbel according 
to formulas (11), (12).

The calculation results are shown in Fig. 3–6 and 
in Table 1, from which follows.

### Table 1

<table>
<thead>
<tr>
<th>№</th>
<th>Electric locomotive type</th>
<th>Energy capacity, ( E_n ), kWh</th>
<th>Electric capacity of the storage unit, ( F )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( M[C_u] )</td>
<td>( \sigma_{(C_u)} )</td>
</tr>
<tr>
<td>1</td>
<td>VL11M6</td>
<td>49.6</td>
<td>45.1</td>
</tr>
<tr>
<td>2</td>
<td>VL8</td>
<td>81.87</td>
<td>108.1</td>
</tr>
</tbody>
</table>

Note: \( V \) is the probability of the corresponding mathematical expectation.

The recuperation voltage, the average in the RB 
phase, and in terms of instantaneous values, obeys 
the Gauss law with a probability according to Pear-
son’s criterion equal to 0.23 (Fig. 3 and 5). Energy 
\( E_n \) and electric \( C \) capacities of on-board storage 
units are distributed exponentially with a clear preva-
ence of minimum values. In this case, between \( C \) 
and \( E_n \), as in formula (2), qualitatively direct pro-
portionality is observed (Fig. 4 and 6). The differ-
e in the possible values of capacity \( C \), given in 
Table 1, indicates that the task of designing an on-
board storage device (in terms of mass and dimensions) is ambiguous, therefore, in each specific case of the type of electric locomotive and RB modes, it must be solved individually, taking into account the probability \( V \) of the corresponding capacity values (see Table 1).

![Fig. 3. Histograms (1) and theoretical laws of instantaneous (a) and average (b) voltage values per a recuperation phase at a current collector of VL11M6 electric locomotive in recuperative braking mode](image)

Fig. 3. Histograms (1) and theoretical laws of instantaneous (a) and average (b) voltage values per a recuperation phase at a current collector of VL11M6 electric locomotive in recuperative braking mode

![Fig. 4. Statistical distribution laws of energy (a) and electric capacity (b) of on-board storage device of VL11M6 electric locomotive](image)

Fig. 4. Statistical distribution laws of energy (a) and electric capacity (b) of on-board storage device of VL11M6 electric locomotive

![Fig. 5. Statistical (1) and theoretical (2) distributions of recuperated voltage of average values per a phase of VL8 electric locomotive in the section Nd–Junction–Chaplino](image)

Fig. 5. Statistical (1) and theoretical (2) distributions of recuperated voltage of average values per a phase of VL8 electric locomotive in the section Nd–Junction–Chaplino
The probabilistic patterns of the influence of the nature of the recuperation energy change on the capacity of the on-board storage in the phase of regenerative braking have been established.

The developed method and the technique based on it, allow evaluating the functional parameters of on-board storage units of all types of electric rolling stock, taking into account the stochastic nature of recuperable voltages and currents. Numerical and graphical dependences of the energy and electric capacities of the on-board storage units are recommended for predicting and evaluating these parameters for various modes of regenerative braking.

Conclusions

1. The main parameters determining the functional capabilities of the on-board capacitive storage units of recuperable electric energy are the maximum energy capacity, the minimum electric capacity and the energy reversal time, which are not calculated for a full trip of the ERS with a train, but only for the phase of its recuperative braking.

2. The most efficient and accurate method for evaluating the recuperation energy, and hence the energy storage capacity, is an experimental calculation method based on the use of time dependences of voltage and current obtained in real recuperation modes.

3. The random nature of the values of the recuperation energy and voltage determines the random nature and capacity of the storage unit, which can be assessed either by the total statistical set of \( W_p \) and \( U_p \), or by Gumbel’s statistics of extreme capacity values, or by determining its most probable value.

4. It has been established that the energy capacity of the on-board storage units of the operated electric locomotives is distributed according to an exponential law with a mathematical expectation of 50...82 kWh, and the absolute values of the capacity in the range of \( x \approx 16,6...28,45 \) F are observed with a probability 0.411 ... 0.235, and in the range of \( \approx 46...49 \) F – with a probability of 0.652...0.6. The maximum values of capacities in the range of \( \approx 72...165 \) F are rare, their probability does not exceed 0.04...0.06.
**LIST OF REFERENCE LINKS**


ЕЛЕКТРИЧНИЙ ТРАНСПОРТ, ЕНЕРГЕТИЧНІ СИСТЕМИ ТА КОМПЛЕКСИ

М. О. КОСТИН1, А. М. МУХА2, О. Г. ШЕЙКІНА3, О. Я. КУРІЛЕНКО4

1 Каф. «Електротехніка та електромеханіка», Дніпропетровський національний університет залізничного транспорту імені академіка В. Лазаряна, вул. Лазаряна, 2, Дніпро, Україна, 49010, тел. +38 (056) 373 15 37, ел. пошта nkostin@ukr.net, ORCID 0000-0002-0856-6397
2 Каф. «Електротехніка та електромеханіка», Дніпропетровський національний університет залізничного транспорту імені академіка В. Лазаряна, вул. Лазаряна, 2, Дніпро, Україна, 49010, тел. +38 (056) 373 15 37, ел. пошта andreimu@i.ua, ORCID 0000-0002-5629-4058
3 Каф. «Електротехніка та електромеханіка», Дніпропетровський національний університет залізничного транспорту імені академіка В. Лазаряна, вул. Лазаряна, 2, Дніпро, Україна, 49010, тел. +38 (056) 373 15 37, ел. пошта sheikina.diit@gmail.com, ORCID 0000-0002-5367-2674
4 Каф. «Електротехніка та електромеханіка», Дніпропетровський національний університет залізничного транспорту імені академіка В. Лазаряна, вул. Лазаряна, 2, Дніпро, Україна, 49010, тел. +38 (056) 373 15 37, ел. пошта kyrilenko@gmail.com, ORCID 0000-0002-0856-6397

Визначення енергетичної та електричної ємностей бортового суперконденсаторного накопичувача енергії регенерації

Мета. У статті передбачено розробку на основі асимптоматичної теорії екстремальних статистик Гумбеля методу визначення основних функціональних параметрів бортового суперконденсаторного накопичувача енергії регенерації з урахуванням стохастичного характеру зміни рекуперованих напруг та струму. Методика. Для досягнення поставленої мети використані методики, приклади та комп’ютерні системи тимчасової реєстрації рекуперованих напруг та струмів на діючих електровозах, методи теорії випадкових процесів і способи ймовірнісно-статистичної обробки рестратограм напруг і струмів. Результати. Запропоновано та практично застосовано експериментально-роздрукуваний метод оцінки енергії рекуперації. Розроблено ймовірнісний метод визначення енергетичної та електричної ємностей бортових суперконденсаторних накопичувачів енергії рекуперації. Використано численні ймовірнісно-статистичні роздруки енергетичної та електричної ємностей бортового накопичувача для електровозів ВЛ8 та ВЛ11М6 під час їх експлуатації на ділянках Придніпровської залізниці. Установлено, що енергетична та електрична ємність бортових накопичувачів розподіляють за експоненційним законом із чітким превалюванням їх мінімальних значень і дотриманням між ними прямої пропорційності. Наукова новизна. Уперше розроблено автономний фазовий режим регенераційного гальмування електрорухомого складу, що дозволяє істотно зменшити масогабаритні показники суперконденсаторного накопичувача. Адаптовано асимптоматичну теорію екстремальних статистик Гумбеля до методики розрахунку енергетичної та електричної ємності бортового накопичувача, що дозволило врахувати стохастичний характер зміни рекуперованих напруг та струму. Установлено ймовірнісний закономірність впливу характеру зміни енергії рекуперації на ємність бортового накопичувача в фазі регенераційного гальмування. Подальший розвиток отримав розрахунково-експериментальний метод оцінки енергії рекуперації, що базується на моніторингу та використанні часових залежностей напруг та струму, отриманих у реальних режимах рекуперативного гальмування. Уперше у системах електричної тяги запропоновано здійснювати перехід від режиму рекуперативного гальмування до режиму рекуперативної регенерації. Практична значимість. Розробленого методу і методики, що базується на ньому, дозволяють оцінювати функціональні параметри бортових накопичувачів усіх видів електрорухомого складу з урахуванням стохастичного характеру рекуперованих напруг і струмів. Чисельно-графічні залежності енергетичної та електричної ємностей бортового накопичувача рекомендовані для прогнозування й оцінки цих параметрів за різних режимів регенераційного гальмування. Оскільки задача конструювання бортового накопичувача (за часою і габаритами) неоднозначна, у кожному конкретному випадку для типу електровоза та режимів рекуперації її потрібно розв’язувати індивідуально з урахуванням імовірності відповідних значень ємності.

Ключові слова: бортовий накопичувач; енергетична ємність; суперконденсатор; енергія рекуперації; випадковий процес; напруга; струм; екстремальна статистика.
REFERENCES


17. Sumpavakup, C., Ratniyomchai, T., & Kulworawanichpong, T. (2017). Optimal energy saving in DC railway system with on-board energy storage system by using peak demand cutting strategy. *Journal of Modern Transportation*, 25(4), 223-235. DOI: https://doi.org/10.1007/s40534-017-0146-6 (in English)

Received: November 13, 2020
Accepted: March 15, 2021