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ІНФОРМАЦІЙНО-КОМУНІКАЦІЙНІ ТЕХНОЛОГІЇ ТА МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ

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MODELLING THE MODIFIED METHOD OF ANALYTIC HIERARCHY PROCESS BY MEANS OF CONSTRUCTIVE AND PRODUCTIVE STRUCTURES

Purpose. In the study it is supposed: 1) to extend the classical method of analytic hierarchy process (AHP) for a great number of alternatives and criteria; 2) to build a model of constructive decision making process using a modified method of analytic hierarchy process with sorting (AHPS). **Methodology.** To achieve this purpose the mechanism of constructive and productive structures (CPS) was used; the refining transformations of the generalized constructive-productive structure (GCPS) were fulfilled. **Findings.** The developed model of the constructive process is the interaction between the three structures: the general CPS of AHPS, which allows to set criteria and alternatives and performs the decomposition of task hierarchical structure; CPS of grouping and sorting, which divides alternatives (criteria) into groups and implements the classic single-level AHP for each group, as well as calculates estimates of paired comparisons based on the input data; CPS of single-level classic AHP, which allows to fill the matrix of paired comparisons and calculates the ranks of alternatives. All three structures interact at different levels of transformations: by data conformity at the level of concretization and using of implementations. The proposed model allowed moving to the more abstract level in presentation of decision making problem solving for a great number of criteria and alternatives. **Originality.** The paper proposes to use CPS mechanism for formalizing modifications of AHP with sorting for decision making problem solving with a great number of criteria and alternatives. **Practical value.** The formalization of the presentation of the analytic hierarchy process and its modifications allows extending the range of applications of this method, as well as unifying the description of various AHP modifications. Such presentation provides the possibility for developing the programs to implement the method hybrid modifications. Using different interpretations presented in the article of CPS will allow for other approaches in determining the coherence of pairwise comparison matrices, estimate calculation and ranks of alternatives and criteria.

Keywords: modelling; constructive and productive structure; constructive process; analytic hierarchy process; modification

Introduction

Analytic Hierarchy Process (AHP) [4, 11], proposed by Saaty, received worldwide recognition and is used to solve the decision-making problems in different areas. There are many versions of this method, which take into account the specificity of the tasks, can reduce the existing restrictions on the use of this method [1–3, 13, 14], or use AHP in combination with other decision-making methods (mathematical methods of multi-criteria analysis, statistical methods etc.) [10, 12]. There are a lot of developed software tools, which implement both the method itself and its modifications [2, 9, 14, 15].

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[14] presents the modification of AHP with sorting (AHPS) which may be used while ranking a large number of alternatives. The essence of this method is that all alternatives are divided into groups in threes (fours) and for each group the classical AHP is applied. If the position of alternatives in groups changes, the rearrangement is performed. Some estimates not yet identified by the expert are calculated on the basis of already determined ones at each step. This greatly facilitates the work of an expert.

Purpose

The purpose of this work is to extend the classical AHP for a great number of alternatives and criteria. To do this, it is proposed to present the AHPS-based constructive decision-making process by the constructive and productive structures (CPS) [6]. In [8] CPS tools formalize the alternatives ranking process using the classical AHP.

To represent AHPS there was developed a system of three interacting CPS: directly AHPS, grouping and sorting CPS and CPS of single-level classical AHP.

Methodology

To achieve this purpose, the mechanism of constructive and productive structures is used. CPS is a powerful device for formalization and modelling of processes [5–8]. By performing different transformations of the generalized constructive and productive structure (GCPS) [6], namely, specialization, interpretation, specification and implementation, the different models are developed [7]. GCPS is called a triple [6]:

$$C_{\mathrm{G}} = \langle M, \Sigma, \Lambda \rangle,$$

where M – heterogeneous structure medium Σ – signature; consisting of sets of the binding operations, substitution and output operations; operations on attributes and substitutive relations; Λ – constructive axiomatics [6].

CPS purpose is to form the sets of structures using binding, substitution and other operations defined by axiomatic rules.

Findings

This paper presents a modified AHPS model [14] on the basis of CPS with unconstrained number of criteria and alternatives.

All three CPS interact at the specification level: data coherence connection and at the implementation level: CPS AHPS uses implementation of grouping and sorting CPS for the criteria and for a set of alternatives for each criterion, the grouping and sorting CPS uses the implementation for each CPS group of a single-level AHP.

Constructive and productive structure of AHPS. Let us determine the GCPS specialization [6] to represent the analytic hierarchy process with sorting:

$$C = \langle M, \Sigma, \Lambda \rangle \;{}_{S}\!\!\mapsto C_{AHPS} \left\langle M_{AHPS}, \Sigma_{AHPS}, \Lambda_{AHPS} \right\rangle$$

where C – ОКПС, M – heterogeneous medium, Σ – signature, Λ – axiomatics, ${}_{S}\!\!\mapsto$ – specialization operation, $\Lambda_{AHPS} = \Lambda \cup \Lambda_1$, $\Lambda_1 = \{M_{AHPS} \supset T_1 \cup N_1$, $\Sigma_{AHPS} = \{\Xi, \Theta, \Phi, \Pi\}$, $\Theta = \{\Rightarrow, |\Rightarrow, \|\Rightarrow\}$, $\Pi = \{\to\}$, $\Phi = \{\div, *, :=, <, \nabla\}\}$, $\Xi = \{\cdot, \circ, \Diamond\}$, Ξ – binding operations, Θ – output operations, Π – substitution operations, Φ – operation on attributes.

Partial axiomatics Λ_1 contains the following definitions, additions and constraints that specify alphabet, medium attributes, substitutive relations, set the features of substitution and output operations.

Terminal alphabet contains a set of alternatives $\{{}_{name,v}x_i\}$ and criteria $\{{}_{name,v}k_p\}$ with their attributes: x_i – alternative identifier, $name$ – semantics, v – global priority (weight); k_p – criterion identifier.

Alternatives and criteria, valid assessment values are contained in a heterogeneous medium M_{AHPS}.

The following operations on attributes are introduced:

$\div(c;n;L)$ – conditional n operations from the list L, if $c = true$, $L = (j_1, j_2, ..., j_n)$, operations are presented in the prefix form;

$*$ – vector-number multiplication operation;

$:=$ – assignment operation;

$<$ – less-than comparison;

∇ – attribute value seek operation, by an external server.

The substitution rules are written as $\psi_{r_{r,i,j}} : \left\langle s_{r,i,j}, g_{r,i,j} \right\rangle \in \Psi$, where $s_{r,i,j}$ – substitu-

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tive relation, $g_{r,i,j}$ – a set of operations on attributes, r – rule number, i, j – numbers of the first and the second pair of alternatives. Three-level indexing is used for ordering the substitution rules.

Binary partial output operation [6] $l^* = (|\Rightarrow(\Psi,l))$ (here l, l^* –forms before and after the substitution operation), consists of:

1) selecting one of the substitution rules $\psi_r : \langle s_{r,i,j}, g_{r,i,j} \rangle \in \Psi$, with substitutive relations $s_{r,i,j}$ and performing the substitution operation on its basis. Availability of substitutive relation $s_{r,i,j}$ is determined by the availability attribute value $d_{r,i,j} \lrcorner s_{r,i,j}$: if $d_{r,i,j} \lrcorner s_{r,i,j} = 1$ the relation is available, $d_{r,i,j} \lrcorner s_{r,i,j} = 0$ – not available; the availability of rules is regulated by operations on attributes or is given by axiomatics;

2) carrying out operations on attributes $g_{r,i,j}$.

The order of the operation on attributes in the process of performing partial output operation is given by the attribute τ_j, where $\tau_j \in I$, $I = \{\tau_0, \tau_1\}$, $I \subset M_{AHPS}$, τ_0 – the operation on attribute is performed before the substitution operation, τ_1 – after the substitution operation.

Complete output (or output) operation is the sequential partial output operation, starting from the initial nonterminal and finishing with the construction that satisfies the output completion condition. The result of the complete output operation is the construct containing the ordered sequence of alternatives.

The output completion condition is the absence of non-terminals in the form.

Suppose we have the following basic algorithmic structure (BAS) [6], which comprises the steps of performing operations by condition, matrix operations, as well as the launch of AHPS for criteria and alternatives:

$$C_{A,AHPS} = \langle M_{A,AHPS}, \Sigma_{A,AHPS}, \Lambda_{A,AHPS} \rangle,$$

where $M_{A,AHPS}$ – heterogeneous medium that contains $V_{A,AHPS}$, $\Sigma_{A,AHPS}$ – signature and $\Lambda_{A,AHPS}$ – axiomatics, $V_{A,AHPS} \supset \{A_1^0 |_{A_i,A_j}^{A_i \cdot A_j}, \; A_{22}^0 |_{a,b,}^{a \circ b}, A_{23}^0 |_{\alpha,b}^{\alpha \lozenge b}\}$ – a set of forming algorithms for a particular server, and $\{A_2 |_{l_h,l_q,f_i}^{f_j}, A_3 |_{f_i,\Psi}^{f_j}, A_4 |_{f_i,\Psi}^{f_j}\} \bigcup V_w$ – a set of constructed algorithms, $\{A_5 |_{c,n,L}^{L}, A_6^0 |_{a,b}^{P}$, $A_7^0 |_{a,b}^{a}, A_8^0 |_{a,b}^{c}, A_9^0 |_{a}^{b}, A_{18}^0 |_{a,b}^{c}$, $A_{10}^0 |_{a,b}^{c}$, $A_{11}^0 |_{a,b}^{c}$, $A_{12}^0 |_{a,b}^{c}$, $A_{13}^0 |_{a,b}^{a}, A_{14}^0 |_{a,b}^{c}, A_{15}^0 |_{{}_N\overset{=}{\gamma}}^{is}, A_{16}^0 |_{{}_N\overset{=}{\gamma}}^{\bar{r}}, A_{17}^0 |_{a}^{c}$, $A_{18}^0 |_{a,b}^{c}$, $A_{19}^0 |_{a,b}^{c}, A_{20}^0 |_{a,b}^{c}, A_{21}^0 |_{x_i,x_j,k_p}^{a,h} \} \in V_w$ – algorithms for operations Φ on attributes.

The above algorithms execute the following operations:

– $A_1^0 |_{A_i,A_j}^{A_i \cdot A_j}$ – algorithm concatenation (sequential algorithm A_i after A_j);

– $A_2 |_{l_h,l_q,f_i}^{f_j}$ – substitution;

– $A_3 |_{f_i,\Psi}^{f_j}, A_4 |_{\sigma,\Psi}^{\bar{\Omega}}$ – partial and complete output.

Here f_i, f_j –forms, σ –initial nonterminal, $\bar{\Omega}$ – a set of formed constructs;

– $A_5 |_{c,n,L}^{L}$ – execution of n algorithms from the list L, if $c = true$;

– $A_6^0 |_{a,b}^{c}$ – calculation of the product $a * b$, a and b can be matrices or numbers;

– $A_7^0 |_{a,b}^{a}$ – assigning a value to a variable $a = b$;

– $A_9 |_{a}^{b}$ – determining the value of a by an external server;

– $A_{10}^0 |_{a,b}^{c}$ – calculation of the quotient of a by b;

– $A_{11}^0 |_{a,b}^{c}$ – calculation of the remainder on dividing a by b;

– $A_8^0 |_{a,b}^{c}, A_{12}^0 |_{a,b}^{c}, A_{13}^0 |_{a,b}^{c}, A_{19}^0 |_{a,b}^{c}, A_{20}^0 |_{a,b}^{c}$ – comparison of numbers a and b, if the condition is satisfied ($a \le b, a \ne b$, $a = b$, $a > b, a < b$), then $c = true$, otherwise $c = false$;

– $A_{13}^0 |_{a,b}^{a}$ – assigning the value b to the variable a, the values a and b can be vectors, matrices, or numbers;

– $A_{14}^0 |_{a,b}^{c}$ – logical AND of the two conditions a and b, true, if both conditions are true;

– $A_{15}^0 |_{{}_N\overset{=}{\gamma}}^{is}$ – calculation of the conformity relation of the pairwise comparison matrix (PCM) ${}_N\overset{=}{\gamma}$;

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– $A_{16}^{0}|_{_N\overline{\overline{\gamma}}}^{\overline{r}}$ – calculation of the alternative priority vector by PCM $_N\overline{\overline{\gamma}}$;

– $A_{17}|_a^c$ – calculation of integral part of the real number a;

– $A_{18}^{0}|_{a,b}^{c}$ – calculation of the sum $a+b$, a and b can be matrices or numbers;

– $A_{21}^{0}|_{x_i,x_j,k_p}^{a,h}$ – determination of the link weight of i and j alternatives by the criterion k_p by an external server ($h=1$);

– $A_{22}^{0}|_{a,b}^{a\circ b}$ –binding alternatives and criteria, where a,b – identifiers of alternatives or criteria or links between them;

– $A_{23}^{0}|_{\alpha,b}^{\alpha\Diamond b}$ – binding b implementation result to non-terminal of CPS implementation use α.

Interpretation of the main CPS for the modified analytic hierarchy process:

$$\left\langle C_{AHPS} = \left\langle M_{AHPS}, \Sigma_{AHPS}, \Lambda_{AHPS} \right\rangle,\right.$$

$$C_{A,AHPS} = \left\langle M_{A,AHPS}, \Sigma_{A,AHPS}, \Lambda_{A,AHPS} \right\rangle \left.\right\rangle {}_I\!\mapsto$$

$$_I\!\mapsto_I C_{AHPS} = \left\langle M_{AHPS}, \Sigma_{AHPS}, \Lambda_{I,AHPS}, Z \right\rangle,$$

where $_I\!\mapsto$ – interpretation operation; Z – a set of servers that can use all BAS algorithms; $C_{A,AHPS}$

$$\Lambda_{I,AHPS} = \Lambda_{AHPS} \cup \Lambda_3,$$

$$\Lambda_3 = \{(A_1^0|_{A_i,A_j}^{A_i\cdot A_j} \lrcorner \cdot), (A_2|_{l_h,l_q,f_i}^{f_j} \lrcorner \Rightarrow), \ (A_3|_{f_i,\Psi}^{f_j} \lrcorner |\Rightarrow),$$

$$A_4|_{\sigma,\Psi}^{\overline{\Omega}} \lrcorner ||\Rightarrow), (A_6^0|_{a,b}^{c} \lrcorner *), (A_7^0|_{a,b}^{a} \lrcorner :=),$$

$$(A_8^0|_{a,b}^{c} \lrcorner <), \ (A_5^0|_{c,n,L}^{L} \lrcorner \div), (A_{23}^0|_{\alpha,b}^{\alpha\Diamond b} \lrcorner \Diamond)\}.$$

Let us represent CPS specification for the analytic hierarchy process with sorting:

$$_I C_{AHPS} = \left\langle M_{AHPS}, \Sigma_{AHPS}, \Lambda_{I,AHPS}, Z \right\rangle \ {}_K\!\mapsto$$

$$_K\!\mapsto C_{K,AHPS} = \left\langle M_{AHPS}, \Sigma_{K,AHPS}, \Lambda_{I,AHPS} \cup \Lambda_4 \cup\right.$$

$$\left.\cup \Lambda_5, Z \right\rangle,$$

where

$$\Lambda_4 = \{T_1 = \{x_1, x_2, x_3, ..., x_N, k_1, ..., k_P\},$$

$$N_1 = \{_{P,N}\sigma, {}_Q\overline{\lambda}, \alpha_1, ..., \alpha_P, {}_Q\overline{\phi_1}, ..., {}_Q\overline{\phi_P}, \chi\},$$

$$U = \{_{P,N}\sigma\}, \quad \Psi_K = \{\psi_r : \left\langle s_{r,i,j}, g_{r,i,j} \right\rangle\}, \ r = \overline{1,6}\},$$

r – rule number, i – number of the first alternative, j – the number of the second alternative of the pair, x_i – terminal for identifier of the i-th alternative, k_l – terminal for identifier of the l-th criterion, U – a set of initial non-terminals, α_l – non-terminal for processing alternatives by the l-th criterion, χ – non-terminal for implementation of the grouping and sorting CPS for criteria, $_Q\overline{\lambda}$ – non-terminal of CPS criteria (where $Q = \{\overline{r}, is, N\}$ – a set of attributes: $\overline{r}$ – vector of PCM priorities, is – matrix conformity relation, N – matrix dimension), $_Q\overline{\phi}_l$ – PCM alternatives by l-th criterion.

Partial axiomatic Λ_5 is as follows.

The number of criteria P and the number of alternatives N, as well as the semantics of alternatives and criteria are given at the stage of execution by an external server.

The record of the sequential concatenation of several terminals, non-terminals and sequential operations on attributes will be represented as follows:

$$x_1 \cdot x_2 \cdot ... \cdot x_n = \prod_{i=1}^{n} x_i .$$

The record $\prod_{i=1}^{N}\prod_{j=i+1}^{N}(\alpha \ \ _{d_{r,i,j}} \rightarrow \beta)$ means that the rule consists of a sequence of substitutive relations with a given availability attribute. If the substitutive relation is available, then it is performed and the availability of the next relation in the sequence is determined, otherwise this relation is omitted, and the availability of the next one in the sequence is determined.

The rules that do not change the current construct have void substitutive relation.

It is assumed that $d_{r,i,j} \lrcorner s_{r,i,j} = 1$, for each $\mathrm{r} = \overline{1,5}$, so this attribute in these rules is omitted. Here are the rules and their brief description.

The substitutive relation $s_{1,0,0}$ is used to enter the processing sequence of criteria and alternatives by criteria. Operations on attributes determine the

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quantity and the semantics of criteria and alternatives:

$$s_{1,0,0} = \left\langle {}_{P,N}\sigma \to \chi|^{Q\bar{\lambda}}_{P\bar{k},0,P\bar{k}} \lozenge_Q \bar{\lambda} \cdot \prod_{p=1}^{P} (\alpha_p) \right\rangle,$$

$$_{\tau_0} g_{1,0,0} = \langle\ N := \nabla(N), P := \nabla(P), \prod_{i=1}^{P} (k_i := \nabla(k_i));$$

$$\prod_{i=1}^{N} (x_i := \nabla(x_i))\ \rangle.$$

The relation $s_{2,0,0}$ uses implementation of the grouping and sorting CPS for the alternatives for each criterion, and $s_{3,0,0}$ is used to get the implementation results of the sorting and grouping CPS for criteria:

$$s_{2,0,0} = \langle\ \prod_{p=1}^{P} (\alpha_p \quad \to \chi|^{Q\bar{\phi}_p}_{N\bar{x},p,P\bar{k}} \lozenge_Q \bar{\phi}_p)\ \rangle.$$

$$s_{3,0,0} = \langle\ \chi|^{Q\bar{\lambda}}_{P\bar{k},0,\varepsilon} \lozenge_Q \bar{\lambda} \to {}_Q\bar{\lambda} \rangle.$$

The relation $s_{4,0,0}$ is used to obtain the implementation result of the grouping and sorting CPS for alternatives:

$$s_{4,0,0} = \langle\ \prod_{p=1}^{P} (\chi|^{Q\phi_p}_{N\bar{x},p,P\bar{k}} \lozenge_Q \bar{\phi}\ _p \to {}_Q\bar{\phi}_p) \rangle.$$

The following substitutive relation is aimed to enter a set of alternatives into the construct. Operations on attributes contain the calculation of global alternative priorities:

$$s_{5,0,0} = \left\langle {}_Q\bar{\lambda} \cdot \prod_{p=1}^{P} ({}_Q\bar{\phi}_p) \to \prod_{i=1}^{N} ({}_v x_i) \right\rangle,$$

$$_{\tau_0} g_{5,0,0} = \left\langle \prod_{i=1}^{N} (v \lrcorner x_i := (\bar{r}_p \lrcorner \lambda) * r_i \lrcorner \phi_p) \right\rangle.$$

The set of substitutive relations $s_{6,0,0}$ allows ordering the alternatives into constructs according to their ranks:

$$s_{6,0,0} = \left\langle \prod_{i=1}^{N-1} \prod_{j=i+1}^{N} ({}_v x_i \cdot {}_v x_j \ {}_{d_{6,i,j}} \to {}_v x_j \cdot {}_v x_i) \right\rangle,$$

$$_{\tau_0} g_{6,0,0} = \left\langle \prod_{i=1}^{N-1} \prod_{j=i+1}^{N} (\div (v \lrcorner x_i < v \lrcorner x_j ; 1; d_{6,i,j} := 1)) \right\rangle.$$

The implementation of this CPS is the set of alternatives ordered in accordance with the calculated ranks.

Constructive and productive structure of alternatives grouping and sorting (CPS of AGS). Let us determine the GCPS specialization to represent the grouping and sorting subsystem for AHPS:

$$C = \langle M, \Sigma, \Lambda \rangle \ {}_S\!\mapsto C_{GSA} \langle M_{GSA}, \Sigma_{GSA}, \Lambda_{GSA} \rangle$$

where $\Lambda_{GSA} = \Lambda \cup \Lambda_6$, $\Lambda_6 = \{M_{GSA} \supset T_2 \cup N_2$, $\Sigma_{GSA} = \{\Xi, \Theta, \Phi, \Pi\}$, $\Pi = \{\to\}$, $\Theta = \{\Rightarrow, |\Rightarrow, ||\Rightarrow\}$, $\Xi = \{\cdot, \circ, \lozenge\}$, $\Phi = \{\div, *, :=, >, \le, \%, +, \backslash, =, \ne, \&, \hbar$, $\lambda, []\}\}$, Ξ – binding operation, Θ – output operations, Φ – operations on attributes, Π – substitution operations.

Partial axiomatics Λ_6 is presented below.

Terminal alphabet contains many alternatives and criteria with their attributes.

The substitution rules include a substitutive relation and a set of operations on attributes. The substitutive relations contain the available attribute d_r, where r – the rule number that takes the value 1 – the relation is available and 0 – not available. For the rules with a constant availability attribute (d_r=1) this attribute is omitted for record simplicity.

To interpret the CPS of alternative grouping and sorting let us use БАС $C_{A,AHPS}$, described above:

$$\langle C_{GSA} = \langle M_{GSA}, \Sigma_{GSA}, \Lambda_{GSA} \rangle,$$

$$C_{A,AHPS} = \langle M_{A,AHPS}, \Sigma_{A,AHPS}, \Lambda_{A,AHPS} \rangle\rangle \ {}_I\!\mapsto$$

$$_I\!\mapsto_I C_{GSA} = \langle M_{GSA}, \Sigma_{GSA}, \Lambda_{I,GSA}, Z_{GSA} \rangle,$$

where

$$\Lambda_{I,GSA} = \Lambda_{GSA} \cup \Lambda_7,\ \Lambda_3 = \{(A_1^0 |^{A_i \cdot A_j}_{A_i, A_j} \lrcorner \cdot),$$

$$(A_2|^{f_j}_{l_h, l_q, f_i} \lrcorner \Rightarrow), (A_3|^{f_j}_{f_i, \Psi} \lrcorner |\Rightarrow),\ A_4 |^{\bar{\Omega}}_{\sigma, \Psi} \lrcorner ||\Rightarrow),$$

$$(A_6^0 |^c_{a,b} \lrcorner *),\ (A_7^0 |^a_{a,b} \lrcorner :=),\ (A_8^0 |^c_{a,b} \lrcorner \le),$$

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$$(A_5^0 |_{c,n,L}^{L} \lrcorner\div),\ (A_{10}^0 |_{a,b}^{c} \lrcorner/),\ (A_{11}^0 |_{a,b}^{c} \lrcorner\%),$$

$$(A_{12}^0 |_{a,b}^{c} \lrcorner\neq),\ (A_{13}^0 |_{a,b}^{c} \lrcorner=),\ (A_{14}^0 |_{a,b}^{c} \lrcorner\&),$$

$$(A_{15}^0 |_{N\bar{\bar{\gamma}}}^{is} \lrcorner\hbar),\ (A_{16}^0 |_{N\bar{\bar{\gamma}}}^{\bar{r}} \lrcorner\lambda\!\!\!^{-}),\ (A_{17} |_{a}^{c} \lrcorner[]),$$

$$(A_{18}^0 |_{a,b}^{c} \lrcorner+),\ (A_{19}^0 |_{a,b}^{c} \lrcorner>),\ (A_{23}^0 |_{\alpha,b}^{\alpha\Diamond b} \lrcorner\Diamond)\}.$$

We concretize CPS of alternative grouping and sorting:

$$_I C_{GSA} = \langle M_{GSA}, \Sigma_{GSA}, \Lambda_{I,GSA}, Z_{GSA} \rangle \ _K\mapsto$$

$$_K\mapsto C_{K,GSA} = \langle M_{GSA}, \Sigma_{K,GSA}, \Lambda_{I,GSA} \cup \Lambda_7 \cup$$

$$\cup \Lambda_8, Z_{GSA} \rangle,$$

where

$$\Lambda_7 = \{T_2 = T_1 \cup \{_{\bar{q}} y_{m,j}\} \cup \{_{\bar{q}} y'_{m,j}\} \cup \{_{\bar{q}} z_{m,j}\},$$

$$m = \overline{1,M} \text{ и } j = \overline{1,4},\ N_2 = \{_{P,N}\chi,\ _{N,p}\bar{\bar{\gamma}},\ _{p,n}\bar{\bar{\beta}},\ _Q\lambda,$$

$$\upsilon |^{-_n L', _Q\lambda}_{_{p,n}\bar{\bar{\beta}}_m, \prod_{i=1}^{n}(_{\bar{q}} y_{m,i}), p, k_p},$$

$$_{k_3,k_4,M}\alpha, \{\chi_m\}\} \cup \{_{ch,n}\Psi_m, _{ch,n}\Psi'_m\},$$

$$m = \overline{1,M},\ \ U = \{_{P,N}\chi\},\ \ \Psi_K = \{\psi_r : \langle s_r, g_r \rangle\},$$

$r = \overline{1,14}\}$, r – rule number, $_{k_3,k_4,M}\alpha$ – is responsible for determining the number of groups with four and three alternatives (k_3, k_4 – number of groups with three or four alternatives in the group, respectively, M – total number of groups); $_{ch,n}\Psi_m$, $_{ch,n}\Psi'_m$ – non-terminals of m-th group of alternatives, with attributes ch – flag indicating the alternative position changes in the group (1 – alternatives changed their position after ranking in the group, 0 – did not change); $_{N,p}\bar{\bar{\gamma}}$ – PCM for N alternatives according to the criterion p, matrix elements, non-terminals $_{a,h}\gamma_{i,j}$ with attributes: a – evaluation of comparison of i and j alternatives, h – evaluation process tool, ($h = 1$ – filled according to the evaluation by an external server, expert, $h = 0$ – without the involvement of an external expert on the basis of substitution rules); $_{p,n}\bar{\bar{\beta}}_m$ – PCM by criterion p for the group m, consisting of n – alternatives, this matrix elements are non-terminals $_{p,a,h}\beta_{m,i,j}$, where the attributes a and h – the same as for $_{a,h}\gamma_{i,j}$; $\upsilon |^{n L', _Q\lambda}_{_{p,n}\bar{\bar{\beta}}_m, \prod_{i=1}^{n}(_{\bar{q}} y_{m,i}), p, k_p}$ – non-terminal of CPS implementation of the single-level classical AHP for the group alternatives: $\prod_{i=1}^{n} {_{\bar{q}} y_{m,i}}$ – set of alternatives for AHP ranking, p – ranking criterion number, $\overline{_P k}$ – criterion vector, $\overline{_Q\lambda}$ – PCM of alternatives of the group with calculated ranks and conformity relation, $_n L'$ – list of alternatives ordered according to the ranks, $_{p,n}\bar{\bar{\beta}}_m$ – PCM of alternatives in the group n; χ_m – non-terminal to prepare m-th group alternatives for ranking; $_{all}\eta$ – non-terminal to calculate the parameters of the general PCM of alternatives (missing evaluations of paired comparisons, conformity relation and matrix completion control); $\{_{\bar{q}} y_{m,i}\}$, $\{_{\bar{q}} y'_{m,i}\}$, $\{_{\bar{q}} z_{m,i}\}$ – set of alternatives in the group m, y, y', z – alternative identifier, $\bar{q} = [name, v, u, \bar{r}, l]$ – set of attributes where $name$ – alternative semantics, v – global priority (weight) of an alternative, u – global number of alternative, $\bar{r}$ – alternative weight vector by criteria, l – criterion number.

The first rule with the substitutive relation, which enters into the construct the sequence of alternatives, PCM by p-th criterion and non-terminal with attributes to work with groups. The operations on attributes calculate the number of groups from 3 and 4 alternatives and the total number of groups. The alternative paired comparison evaluations are completed with default values:

$$s_1 = \left\langle \chi |^{_Q\bar{\lambda}}_{_N\bar{x}, p, _P\bar{k}} \to \prod_{u=1}^{N}(_{name,v}x_u) \cdot {_{p,N}\bar{\bar{\gamma}}} \cdot {_{k_3,k_4,M}\alpha} \right\rangle,$$

$$_{\tau_0}g_1 = \langle flag := 0,\ \div(N\%4 = 0; 3; (k_4 \lrcorner\alpha := N/4),$$

$$(k_3 \lrcorner\alpha := 0), (flag := 1)), \div(N = 3; 3; (k_4 \lrcorner\alpha := 0),$$

$$(k_3 \lrcorner\alpha := 1), (flag := 1)),\ \div(N = 5; 3; (k_4 \lrcorner\alpha := 0),$$

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$$(k_3 \lrcorner \alpha := 2), (flag := 1)), \div (flag = 0;2;$$

$$(k_4 \lrcorner \alpha := N/4 - (3 - N\%4)),$$

$$(k_3 \lrcorner \alpha := (N - k_4 \lrcorner \alpha * 4)/3)),$$

$$M \lrcorner \alpha := k_3 \lrcorner \alpha + k_4 \lrcorner \alpha;$$

$$\prod_{i=1}^{N} (\prod_{j=i+1}^{N} (a \lrcorner {}_p\gamma_{i,j} := 0; a \lrcorner {}_p\gamma_{j,i} := 0; h \lrcorner {}_p\gamma_{i,j} := 0;$$

$$h \lrcorner {}_p\gamma_{j,i} := 0;); a \lrcorner {}_p\gamma_{i,i} := 1; h \lrcorner {}_p\gamma_{i,i} := 0;) \ \rangle .$$

The following relation is applied for breakdown of the alternatives into the groups. The operations on attributes get the conformity between the general list of alternatives and alternatives in groups:

$$s_2 = \left\langle \prod_{u=1}^{N} ({}_{\bar q}x_u) \cdot {}_{k_3,k_4,M}\alpha \rightarrow \prod_{m=1}^{M \lrcorner \alpha} ({}_{ch,n}\psi \cdot \prod_{i=1}^{n \lrcorner \psi_m} {}_{\bar q}y_{m,i}) \right\rangle$$

$${}_{\tau_0}g_2 = \langle \ \prod_{i=1}^{k_3} (n \lrcorner \psi_m = 3;$$

$$\prod_{j=1}^{n \lrcorner \psi_m} (name \lrcorner y_{i,j} := name \lrcorner x_{i*j};$$

$$u \lrcorner y_{i,j} := u \lrcorner x_{i*j})); \prod_{i=k_3+1}^{M} (n \lrcorner \psi_m = 4;$$

$$\prod_{j=1}^{n \lrcorner \psi_m} (name \lrcorner y_{i,j} := name \lrcorner x_{i*j}; u \lrcorner y_{i,j} := u \lrcorner x_{i*j})) \ \rangle .$$

The operations on attributes of s_3 relation determine the attribute values for PCM elements of the alternatives:

$$s_3 = \langle \ \prod_{m=1}^{M \lrcorner \alpha} ({}_{ch,n}\psi \cdot \prod_{i=1}^{n \lrcorner \psi_m} {}_{\bar q}y_{m,i}) \cdot {}_{p,N}\overline{\overline{\gamma}})_{d_3} \rightarrow$$

$$\rightarrow \prod_{m=1}^{M \lrcorner \alpha} ({}_{ch,n}\psi \cdot \prod_{i=1}^{n \lrcorner \psi_m} {}_{\bar q}y_{m,i} \cdot {}_{p,n \lrcorner \psi_m}\overline{\overline{\beta}}_m \cdot \rho_m) \cdot {}_{p,N}\overline{\overline{\gamma}}) \ \rangle ,$$

$${}_{\tau_0}g_3 = \langle \ \prod_{m=1}^{M} (\prod_{i=1}^{n \lrcorner \psi_m} (\prod_{j=1}^{n \lrcorner \psi_m} (a \lrcorner \beta_{m,i,j} := a \lrcorner \gamma_{u \lrcorner y_{m,i}, u \lrcorner y_{m,j}};$$

$$h \lrcorner \beta_{m,i,j} := h \lrcorner \gamma_{u \lrcorner y_{m,i}, u \lrcorner y_{m,j}}))) \ \rangle .$$

The following substitutive relation is used for CPS implementation of the classic single-level AHP for each alternative group:

$$s_4 = \langle \ \prod_{m=1}^{M \lrcorner \alpha} ({}_{ch,n}\psi \cdot \prod_{i=1}^{n \lrcorner \psi_m} {}_{\bar q}y_{m,i} \cdot {}_{p,n \lrcorner \psi_m}\overline{\overline{\beta}}_m \cdot \rho_m)) \rightarrow$$

$$\rightarrow \prod_{m=1}^{M} ({}_{ch,n}\psi \cdot \prod_{i=1}^{n \lrcorner \psi_m} ({}_{\bar q}y_{m,i}) \cdot \upsilon \,|\, ^{\prod_{i=1}^{n \lrcorner \psi_m} ({}_{\bar q}z_{m,i}),{}_Q\lambda}_{{}_{p,n \lrcorner \psi_m}\overline{\overline{\beta}}_m, \prod_{i=1}^{n \lrcorner \psi_m} ({}_{\bar q}y_{m,i}),p,k_p} \ )\rangle .$$

The operations on attributes of the following rule supplement the general PCM with new evaluations:

$$s_5 = \langle \ \prod_{m=1}^{M} (\upsilon \,|\, ^{{}_{p,n \lrcorner \psi_m}\overline{\overline{\beta}}_m, \prod_{i=1}^{n \lrcorner \psi_m} ({}_{\bar q}z_{m,i}),{}_Q\lambda_m}_{{}_{p,n \lrcorner \psi_m}\overline{\overline{\beta}}_m, \prod_{i=1}^{n \lrcorner \psi_m} ({}_{\bar q}y_{m,i}),p,k_p} \ ) \cdot {}_{p,N}\overline{\overline{\gamma}} \rightarrow$$

$$\rightarrow \prod_{m=1}^{M} (\prod_{i=1}^{n \lrcorner \psi_m} {}_{\bar q}z_{m,i},{}_Q\lambda_m) \cdot {}_{p,N}\overline{\overline{\gamma}} \cdot {}_{all}\eta \ \rangle ,$$

$${}_{\tau_1}g_5 = \langle \ \prod_{m=1}^{M} (\prod_{i=1}^{n \lrcorner \psi_m} (\prod_{j=1}^{n \lrcorner \psi_m} (a \lrcorner \gamma_{u \lrcorner z_{m,i}, u \lrcorner z_{m,j}} := a \lrcorner \beta_{m,i,j};$$

$$h \lrcorner \gamma_{u \lrcorner z_{m,i}, u \lrcorner z_{m,j}} := h \lrcorner \beta_{m,i,j}))) \ \rangle .$$

The substitutive relation is used to calculate the general PCM elements by transitiveness and to count the uncompleted elements:

$$s_6 = \langle \ \rangle ,$$

$${}_{\tau_1}g_6 = \langle \ all \lrcorner \eta := 0; \prod_{i=1}^{N} \prod_{j=i+1}^{N} (\prod_{c=1}^{N} (\div (h \lrcorner \gamma_{i,j} = 0;$$

$$4; (\div ((a \lrcorner \gamma_{c,j} \neq 0) \ \& \ (a \lrcorner \gamma_{i,c} \neq 0); 2;$$

$$(sum := sum + a \lrcorner \gamma_{i,c} * a \lrcorner \gamma_{c,j}; q := q + 1));$$

$$a \lrcorner \gamma_{i,j} := sum/q; h \lrcorner \gamma_{i,j} := 0; h \lrcorner \gamma_{j,i} :=$$

$$= 0; a \lrcorner \gamma_{j,i} := \frac{1}{a \lrcorner \gamma_{i,j}}));) \div$$

$$\div (a \lrcorner \gamma_{i,j} = 0; 1; all \lrcorner \eta := all \lrcorner \eta + 1)) \ \rangle .$$

s_7 – substitutive relation for comparison of the alternatives in groups after AHP application. If the order of the alternatives in the group is changed, the corresponding attribute is set to one:

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$$s_7 = \langle \prod_{m=1}^{M} ({}_{ch,n}\psi \cdot \prod_{i=1}^{n\lrcorner\psi_m} ({}_{\bar{q}}\, y_{m,i}) \cdot \prod_{i=1}^{n\lrcorner\psi_m} ({}_{\bar{q}}\, z_{m,i}),_Q \bar{\lambda}_m) \to$$

$$\to \prod_{m=1}^{M} ({}_{ch,n}\psi \cdot \prod_{i=1}^{n\lrcorner\psi_m} ({}_{\bar{q}}\, y_{m,i}) \cdot \prod_{i=1}^{n\lrcorner\psi_m} ({}_{\bar{q}}\, z_{m,i}),_Q \bar{\lambda}_m);$$

$${}_{\tau_0} g_7 = \left\langle \prod_{m=1}^{M} (ch\lrcorner\psi_m := 0; \right.$$

$$\prod_{i=1}^{n\lrcorner\psi_m} (\div(name\lrcorner y_{m,i} = name\lrcorner z_{m,i};$$

$$3; (v\lrcorner y_{m,i} := v\lrcorner z_{m,i}; name\lrcorner y_{m,i} = name\lrcorner z_{m,i};$$

$$u\lrcorner y_{m,i} = u\lrcorner z_{m,i})); \div(name\lrcorner y_{m,i} \neq name\lrcorner z_{m,i};$$

$$1; ch\lrcorner\psi_m := 1 \ \rangle.$$

The relation s_8 is used to calculate the priority vector and the conformity relation for general matrix, if the position of the alternatives in the groups has not changed:

$$s_8 = \langle \prod_{m=1}^{M} ({}_{ch,n}\psi \cdot \prod_{i=1}^{n\lrcorner\psi_m} ({}_{\bar{q}}\, y_{m,i}) \cdot \prod_{i=1}^{n\lrcorner\psi_m} ({}_{\bar{q}}\, z_{m,i}),_Q \bar{\lambda}_m) \times$$

$$\times {}_{p,N}\bar{\bar{\gamma}} \cdot {}_{all}\eta \ {}_{d_8}\!\to {}_Q\bar{\lambda} \rangle,$$

$${}_{\tau_0} g_8 = \langle \ f := 0; \prod_{m=1}^{M} (\div(ch\lrcorner\psi_m = 1; 1; f := 1));$$

$$\div(f = 1; 1; d_9 := 1);$$

$$\div(((f = 0) \ \& \ (all\lrcorner\eta = 0); 1; d_8 := 1); \rangle$$

$${}_{\tau_1} g_8 = \left\langle is\lrcorner\bar{\lambda} := \hbar({}_{p,N}\bar{\bar{\gamma}}); \ \bar{r}\lrcorner\bar{\lambda} := \lambda({}_{p,N}\bar{\bar{\gamma}}) \right\rangle.$$

The substitutive relation s_9 is used to regroup the alternatives. Operations on attributes allow setting the alternative attributes in the new groups:

$$s_9 = \left\langle \prod_{m=1}^{M} ({}_{ch,n}\psi \cdot \prod_{i=1}^{n\lrcorner\psi_m} ({}_{\bar{q}}\, y_{m,i}) \times \right.$$

$$\times \prod_{i=1}^{n\lrcorner\psi_m} ({}_{\bar{q}}\, z_{m,i}),_Q \bar{\lambda}_m) \cdot {}_{p,N}\bar{\bar{\gamma}} \times$$

$$\times {}_{all}\eta \ {}_{d_8}\!\to \prod_{m=1}^{M-1} ({}_{ch,n}\psi'_m \cdot \prod_{i=1}^{n\lrcorner\psi_m} ({}_{\bar{q}}\, y_{m,i}) \times$$

$$\left. \times {}_{p,n\lrcorner\psi'_m}\bar{\bar{\beta}}_m \cdot \rho_m) \cdot {}_{p,N}\bar{\bar{\gamma}} \right\rangle;$$

$${}_{\tau_0} g_9 = \left\langle \prod_{m=1}^{M-1} (l := [\frac{n\lrcorner\psi_m}{2}]; n\lrcorner\psi'_m := l + 2; \right.$$

$$\prod_{i=1}^{l} (name\lrcorner y'_{m,i} := name\lrcorner z_{m,l+i};$$

$$p\lrcorner y'_{m,i} := p\lrcorner z_{m,l+i}; v\lrcorner y'_{m,i} := v\lrcorner z_{m,l+i};$$

$$u\lrcorner y'_{m,i} := u\lrcorner z_{m,l+i}); \prod_{i=1}^{2} (name\lrcorner y'_{m,i+l} := name\lrcorner z_{m+1,i};$$

$$p\lrcorner y'_{m,i+l} := p\lrcorner z_{m+1,i}; v\lrcorner y'_{m,i+l} := v\lrcorner z_{m+1,i};$$

$$u\lrcorner y'_{m,i+l} := u\lrcorner z_{m+1,i});$$

$${}_{\tau_1} g_9 = \langle \prod_{m=1}^{M-1} (\prod_{i=1}^{n\lrcorner\psi'_m} (\prod_{j=1}^{n\lrcorner\psi'_m} (a\lrcorner\beta_{m,i,j} := a\lrcorner\gamma_{u\lrcorner y_{m,i}, u\lrcorner y_{m,j}};$$

$$h\lrcorner\beta_{m,i,j} := h\lrcorner\gamma_{u\lrcorner y_{m,i}, u\lrcorner y_{m,j}}))) \ \rangle.$$

The substitutive relation s_{10} is for implementation of classic AHP for new alternative groups:

$$s_{10} = \langle \prod_{m=1}^{M-1} ({}_{ch,n}\psi'_m \cdot \prod_{i=1}^{n\lrcorner\psi_m} ({}_{\bar{q}}\, y'_{m,i}) \cdot {}_{p,n\lrcorner\psi'_m}\bar{\bar{\beta}}_m \cdot \rho_m) \to$$

$$\to \prod_{m=1}^{M-1} \times$$

$$\times \left({}_{ch,n}\psi' \cdot \prod_{i=1}^{n\lrcorner\psi_m} ({}_{\bar{q}}\, y'_{m,i}) \cdot \upsilon \Big|_{{}_{p,n\lrcorner\psi'_m}\bar{\bar{\beta}}_m, \prod_{i=1}^{n\lrcorner\psi'_m} ({}_{\bar{q}}\, y'_{m,i}), p, k_p}^{\prod_{i=1}^{n\lrcorner\psi'_m} ({}_{\bar{q}}\, z_{m,i}),_Q \lambda} \right).$$

The following rule contains the substitutive relation to get the AHP ranking result in each group and to save the evaluation entered into the general PCM by the expert:

$$s_{11} = \langle \prod_{m=1}^{M-1} (\upsilon \Big|_{{}_{p,n\lrcorner\psi'_m}\bar{\bar{\beta}}_m, \prod_{i=1}^{n\lrcorner\psi'_m} ({}_{\bar{q}}\, y'_{m,i}), p, k_p}^{{}_{p,n\lrcorner\psi'_m}\bar{\bar{\beta}}_m, \prod_{i=1}^{n\lrcorner\psi'_m} ({}_{\bar{q}}\, z_{m,i}),_Q \lambda_m}) \cdot {}_{p,N}\bar{\bar{\gamma}} \to$$

$$\to \prod_{m=1}^{M-1} (\prod_{i=1}^{n\lrcorner\psi'_m} ({}_{\bar{q}} z_{m,i}),_Q \bar{\lambda}_m) \cdot {}_{p,N}\bar{\bar{\gamma}} \cdot {}_{all}\eta \ \rangle,$$

$$_{\tau_1}g_{11}=\langle \prod_{m=1}^{M-1}(\prod_{i=1}^{n\lrcorner\psi'_m}(\prod_{j=1}^{n\lrcorner\psi'_m}(a\lrcorner\gamma_{u\lrcorner z_{m,i},u\lrcorner z_{m,j}}:=a\lrcorner\beta_{m,i,j};$$

$$h\lrcorner\gamma_{u\lrcorner z_{m,i},u\lrcorner z_{m,j}}:=h\lrcorner\beta_{m,i,j})));d_6:=1;d_{12}:=1\rangle.$$

Operations on attributes of the following rule allow determining the changes in the positions of alternatives in the groups after AHP application:

$$s_{12}=\langle\ \rangle,\ _{\tau_0}g_{12}=\langle \prod_{m=1}^{M-1}(ch\lrcorner\psi'_m:=0;$$

$$\prod_{i=1}^{n\lrcorner\psi'_m}(\div(name\lrcorner y'_{m,i}=name\lrcorner z_{m,i};1;v\lrcorner y'_{m,i}:=v\lrcorner z_{m,i}),$$

$$\div(name\lrcorner y'_{m,i}\neq name\lrcorner z_{m,i};1;ch\lrcorner\psi'_m:=1\rangle.$$

The substitutive relation s_{13} is used to calculate the priority vector and the conformity relation for general matrix, if the position of the alternatives in the groups has not changed:

$$s_{13}=\left\langle\prod_{m=1}^{M-1}({}_{ch,n}\psi'\cdot\prod_{i=1}^{n\lrcorner\psi'_m}({}_{\bar{q}}\,y'_{m,i})\cdot\prod_{i=1}^{n\lrcorner\psi'_m}({}_{\bar{q}}\,z_{m,i}),_Q\bar{\lambda}_m)\times\right.$$

$$\left.\times_{p,N}\bar{\bar{\gamma}}\cdot{}_{all}\eta\ {}_{d_{13}}\rightarrow{}_Q\bar{\lambda}\right\rangle,$$

$$_{\tau_0}g_{13}=\left\langle f:=0;\prod_{m=1}^{M-1}(\div(ch\lrcorner\psi'_m=1;1;f:=1));\right.$$

$$\div(f=1;1;d_{13}:=1);$$

$$\left.\div(((f=0)\ \&\ (all\lrcorner\eta=0);1;d_{14}:=1)\ \right\rangle.$$

$$_{\tau_1}g_{13}=\left\langle is\lrcorner\lambda:=\hbar({}_{p,N}\bar{\bar{\gamma}});\ \bar{r}\lrcorner\lambda:=\lambda({}_{p,N}\bar{\bar{\gamma}})\right\rangle.$$

The following substitutive relation is used to restore alternatives in the groups, if their position has changed:

$$s_{14}=\langle \prod_{m=1}^{M}({}_{ch,n}\psi\cdot\prod_{i=1}^{n\lrcorner\psi_m}({}_{\bar{q}}\,y_{m,i}))\cdot(\prod_{m=1}^{M-1}({}_{ch,n}\psi'_m\times$$

$$\times\prod_{i=1}^{n\lrcorner\psi'_m}({}_{\bar{q}}\,y'_{m,i})\cdot\prod_{i=1}^{n\lrcorner\psi'_m}({}_{\bar{q}}\,z_{m,i}),_Q\lambda_m)\ {}_{d_{14}}\rightarrow$$

$$\rightarrow\prod_{m=1}^{M}({}_{ch,n}\psi\cdot\prod_{i=1}^{n\lrcorner\psi_m}{}_{\bar{q}}\,y_{m,i})\ \rangle,$$

$$_{\tau_0}g_{14}=\left\langle\prod_{m=1}^{M-1}(l:=[\frac{n\lrcorner\psi_m}{2}];\right.$$

$$\prod_{i=1}^{l}(name\lrcorner y_{m,l+i}:=name\lrcorner z_{m,i};$$

$$p\lrcorner y_{m,l+i}:=p\lrcorner z_{m,i};v\lrcorner y_{m,l+i}:=v\lrcorner z_{m,i};$$

$$u\lrcorner y_{m,l+i}:=u\lrcorner z_{m,i});\prod_{i=1}^{2}(name\lrcorner y_{m+1,i}:=name\lrcorner z_{m,i+l};$$

$$p\lrcorner y_{m+1,i}:=p\lrcorner z_{m,i+l};v\lrcorner y_{m+1,i}:=.$$

$$=v\lrcorner z_{m,i+l};u\lrcorner y_{m+1,i}:=u\lrcorner z_{m,i+l})$$

$$_{\tau_1}g_{14}=\langle d_3:=1;d_{14}:=0\rangle.$$

Implementation of CPS of alternative grouping and sorting is the non-terminal with calculated alternative rank attributes and the conformity relation for PCM of the alternatives.

Constructive and productive structure for classical single-level AHP. CPS of classical single-level AHP implements completing by an external expert of some paired comparison evaluations, finding the proper number of the matrix, conformity relation of PCM and alternative ranks.

Let us determine GCPC specialization to represent classical single-level AHP:

$$C=\langle M,\Sigma,\Lambda\rangle\ {}_S\mapsto C_{AHP}\langle M_{AHP},\Sigma_{AHP},\Lambda_{AHP}\rangle,$$

where $\Lambda_{AHP}=\Lambda\cup\Lambda_9$, $\Lambda_9=\{M_{AHP}\supset T_3\bigcup N_3$, $\Sigma_{AHP}=\{\Xi,\Theta,\Phi\}$, $\Theta=\{\Rightarrow,|\Rightarrow,||\Rightarrow,\rightarrow\}$, $\Xi=\{\cdot,\circ\}$, $\Phi=\{\div,:=,/,\leq,>,=,\hbar,\lambda,\triangleright\}\}$.

The operation $\triangleright(x_i,x_j,k_p)$ allows setting a value of the link weight between *i* and *j* alternatives (criteria) by k_p criterion, $\triangleright(x_i,x_j,\varepsilon)$ allows setting a value of the link weight between criteria. These operations are executed by an external server.

Terminal alphabet contains many alternatives and criteria with their attributes.

The output process forms the construct that will include the following forms: $_{a,h}(x_i\circ x_j\circ k_p)$ – link of i and j alternatives by criterion k_p, a – link weight, h –attribute, responsible for the weight value derivation process ($h=1$ – filled according to the assessment by an external server expert, $h=0$ – without the involvement of an external expert on the basis of substitution rules); ${}_{a,h}(x_i \circ x_j \circ \varepsilon)$ – link of i and j alternative (criterion) if a comparison criterion is not given; ${}_{Q}\overline{\lambda}$ – pairwise comparison matrix for alternatives; $\prod_{i=1}^{N}({}_{r}x_i)$ – sorted alternative sequence.

For interpretation of this CPS we use the BAS $C_{A,AHPS}$ described above:

$$\left\langle C_{AHP} = \left\langle M_{AHP}, \Sigma_{AHP}, \Lambda_{AHP} \right\rangle, C_{A,AHPS} = \right.$$

$$\left. = \left\langle M_{A,AHPS}, \Sigma_{A,AHPS}, \Lambda_{A,AHPS} \right\rangle \right\rangle {}_{I}\mapsto$$

$${}_{I}\mapsto {}_{I,C_{A,AHPS}} C_{AHP} = \left\langle M_{AHP}, \Sigma_{AHP}, \Lambda_{I,AHP}, Z \right\rangle,$$

where

$$\Lambda_{I,AHP} = \Lambda_{AHP} \cup \Lambda_{11},\ \Lambda_{11} = \{(A_1^0 |_{A_i,A_j}^{A_i \cdot A_j} \lrcorner \cdot),$$

$$(A_2|_{l_h,l_q,f_i}^{f_j} \lrcorner \Rightarrow),\ A_3 |_{f_i,\Psi}^{f_i} \lrcorner |\Rightarrow),\ A_4 |_{\sigma,\Psi}^{\bar{\Omega}} \lrcorner ||\Rightarrow),$$

$$(A_7^0 |_{a,b}^{a} \lrcorner :=),\ (A_8^0 |_{a,b}^{c} \lrcorner \le),\ (A_5^0 |_{c,n,L}^{L} \lrcorner \div),$$

$$(A_{10}^0 |_{a,b}^{c} \lrcorner /), (A_{12}^0 |_{a,b}^{c} \lrcorner \ne), (A_{13}^0 |_{a,b}^{c} \lrcorner =)$$

$$(A_{19}^0 |_{a,b}^{c} \lrcorner >),$$

$$(A_{15}^0 |_{{}_{N}\overline{\gamma}}^{is} \lrcorner \hbar), (A_{16}^0 |_{{}_{N}\overline{\gamma}}^{\bar{r}} \lrcorner \hat{\lambda}), (A_{21}^0 |_{a,b}^{c} \lrcorner \triangleright),$$

$$(A_{22}^0 |_{a,b}^{a \circ b} \lrcorner \circ) \}.$$

Let us perform specification of the interpreted CPS for a single-level AHP:

$${}_{I,C_{A,AHP}} C_{AHP} = \left\langle M_{AHP}, \Sigma_{AHP}, \Lambda_{I,AHP}, Z \right\rangle {}_{K}\mapsto$$

$$C_{K,AHP} = \left\langle M_{AHP}, \Sigma_{K,AHP}, \Lambda_{I,AHP} \cup \Lambda_9 \cup \Lambda_{10}, z \right\rangle,$$

where

$$\Lambda_4 = \{T_3 = T_1,\ N_3 = \{{}_{N}\xi, {}_{P}\rho, \delta, \mu, {}_{P,N}\upsilon, {}_{Q}\overline{\lambda}, {}_{p,N}\overline{\overline{\beta}}\},$$

$$U = \{{}_{P,N}\upsilon\},\ \Psi_K = \{\psi_r : \left\langle s_{r,i,j}, g_{r,i,j} \right\rangle\},\ r = \overline{1,12}\},$$

r – rule number, i and j – number of the first and second pairs of alternatives, U – set of initial non-terminals, ${}_{N}\xi$ – non-terminal to indicate alternative links, ${}_{P}\rho$ – non-terminal to indicate criteria links, $\beta_{p,i,j}$ – non-terminals to indicate links between alternatives i, j by p-th criterion (for simplicity indicated as matrix ${}_{p,N}\overline{\overline{\beta}}$), ${}_{Q}\overline{\lambda}$ – pairwise comparison matrix for alternatives (where $Q = [\overline{v}, is, N]$ – vector of attributes: $\overline{v}$ – vector of PCM priorities, is – matrix conformity relation, N – the number of alternatives).

The substitutive relation $s_{1,0,0}$ serves to change PCM completion and ranking of alternatives x_i by criterion k_p. All axiom input parameters are added to the medium.

$$s_{1,0,0} = \left\langle\ \upsilon|_{{}_{p,N}\overline{\overline{\beta}}\ ,{}_{N}\overline{x}, p, {}_{P}\overline{k}}^{{}_{p,N}\overline{\overline{\beta}}\ ,\prod_{i=1}^{N}({}_{q}^{-}x_i)} \rightarrow {}_{p,N}\overline{\overline{\beta}}\ \cdot \delta \cdot \mu\ \right\rangle.$$

The following substitutive relation is used, if a ranking criterion is specified:

$$s_{2,0,0} = \left\langle\ {}_{p,N}\overline{\overline{\beta}}\ \cdot \delta\ {}_{d_{2,0,0}}\!\!\rightarrow \prod_{i=1}^{N}\prod_{j=1}^{N}({}_{p}\beta_{i,j} \cdot {}_{k_p}\rho) \right\rangle,$$

$${}_{\tau_0} g_{2,0,0} = \left\langle \div(p \lrcorner \overline{\overline{\beta}} > 0; 1; d_{2,0,0} := 1), \right.$$

$$\left. \div(p \lrcorner \overline{\overline{\beta}} = 0; 1; d_{3,0,0} := 1) \right\rangle.$$

The substitutive relation $s_{3,0,0}$ is used to form PCM alternatives, if a criterion is not specified:

$$s_{3,0,0} = \left\langle\ {}_{p,N}\overline{\overline{\beta}}\ \cdot \delta\ {}_{d_{3,0,0}}\!\!\rightarrow \prod_{i=1}^{N}\prod_{j=1}^{N}({}_{p}\beta_{i,j}) \right\rangle.$$

The following rule contains the substitutive relation to determine a connection between the alternatives. Operations on attributes determine the evaluation of alternatives links:

$$s_{4,0,0} = \left\langle \prod_{i=1}^{N}\prod_{j=1}^{N}(\beta_{\ i,j} \rightarrow (x_i \circ x_j \circ \varepsilon) \cdot \beta_{i,j} \right\rangle,$$

$${}_{\tau_1} g_{4,0,0} = \left\langle a \lrcorner (x_i \circ x_j \circ \varepsilon) := a \lrcorner \beta_{i,j}, \right.$$

$$\left. h \lrcorner (x_i \circ x_j \circ \varepsilon) := h \lrcorner \beta_{i,j} \right\rangle.$$

The rules for setting the alternative pairwise comparison values by an expert, where $i = \overline{1,N}$

and $j=\overline{1+i,N}$:

$$s_{5,i,j}=\langle\ \rangle,\ {}_{\tau_1}g_{5,i,j}=\langle a\lrcorner(x_i \circ x_j \circ \varepsilon):=\triangleright(x_i,x_j,\varepsilon),$$
$$a\lrcorner\beta_{i,j}:=h\lrcorner\ (x_i \circ x_j \circ \varepsilon);h\lrcorner\ (x_j \circ x_i \circ \varepsilon):=1,$$
$$a\lrcorner(x_j \circ x_i \circ \varepsilon):=1/(a\lrcorner(x_i \circ x_j \circ \varepsilon)),$$
$$a\lrcorner\beta_{j,i}:=a\lrcorner(x_j \circ x_i \circ \varepsilon);h\lrcorner\beta_{i,j}:=h\lrcorner\ (x_j \circ x_i \circ \varepsilon)\ \rangle.$$

The relation for PCM completion check. If the matrix is completed, then based on the operations on attributes we calculate the conformity relation and fill the alternative priority vector:

$$s_{6,0,0}=\left\langle \begin{array}{l}\prod_{i=1}^{N}\prod_{j=1}^{N}(x_i \circ x_j \circ \varepsilon)\ {}_{d_{6,0,0}}\!\!\rightarrow \\ \rightarrow \prod_{i=1}^{N}\prod_{j=1}^{N}(x_i \circ x_j \circ \varepsilon)\cdot {}_{Q}\overline{\lambda})\end{array}\right\rangle$$

$${}_{\tau_0}g_{6,0,0}=\langle d_{6,0,0}:=1, full:=1,$$
$$\prod_{i=1}^{N}\prod_{j=i+1}^{N}(\div(a\lrcorner(x_i \circ x_j \circ \varepsilon)\le 0;1;$$
$$full:=0)),\div(full=0;1;d_{6,0,0}:=0)\rangle,$$
$${}_{\tau_1}g_{6,0,0}=\langle is\lrcorner\overline{\lambda}:=\hbar({}_{p,N}\overline{\overline{\beta}});\ \overline{r}\lrcorner\overline{\lambda}:=\lambda({}_{p,N}\overline{\overline{\beta}})\rangle.$$

The relation $s_{7,0,0}$ enters the sequence of alternatives with the weights into the construct, if PCM conformity relation is valid:

$$s_{7,0,0}=\left\langle \begin{array}{l}\prod_{i=1}^{N}\prod_{j=1}^{N}(x_i \circ x_j \circ \varepsilon)\times \\ \times {}_{Q}\overline{\lambda}\cdot\mu\ {}_{d_7}\!\!\rightarrow \prod_{i=1}^{N}({}_{\overline{q}}x_i)\cdot {}_{Q}\overline{\lambda}\end{array}\right\rangle$$

$${}_{\tau_0}g_{7,0,0}=\langle\div((is\lrcorner\overline{\lambda})\le 0,011;\ 1;d_{7,0,0}:=1)\rangle,$$
$${}_{\tau_1}g_{7,0,0}=\langle\ \prod_{i=1}^{N}(v\lrcorner x_i:=r_i\lrcorner\overline{\lambda});d_{7,0,0}:=0\rangle.$$

The following set of rules ($i=\overline{1,N-1}$, $j=\overline{i+1,N}$) defines the relations for descending ordering of alternatives according to their weights:

$$s_{8,i,j}=\langle({}_{v}x_i\cdot{}_{v}x_j)\ {}_{d_{8,i,j}}\!\!\rightarrow({}_{v}x_j\cdot{}_{v}x_i)\rangle,$$
$${}_{\tau_0}g_{8,i,j}=\langle\div(v\lrcorner x_j>v\lrcorner x_i;1;d_{8,i,j}:=1)\rangle.$$

The substitutive relation $s_{9,0,0}$ is used to establish the link between the alternatives by the given criteria:

$$s_{9,0,0}=\left\langle\prod_{i=1}^{N}\prod_{j=1}^{N}(\beta_{i,j}\cdot{}_{k_p}\rho\rightarrow(x_i \circ x_j \circ k_p)\cdot\beta_{i,j}\right\rangle,$$
$${}_{\tau_1}g_{9,0,0}=\langle a\lrcorner(x_i \circ x_j \circ k_p):=a\lrcorner\beta_{i,j},$$
$$h\lrcorner(x_i \circ x_j \circ k_p):=h\lrcorner\beta_{i,j}\rangle.$$

The rules for setting the alternative pairwise comparison values by an expert by p-th criterion, where $i=\overline{1,N}$ and $j=\overline{1+i,N}$:

$$s_{10,i,j}=\langle\ \rangle,$$
$${}_{\tau_1}g_{10,i,j}=\langle a\lrcorner(x_i \circ x_j \circ k_p):=\triangleright(x_i,x_j,k_p),$$
$$a\lrcorner\beta_{i,j}:=a\lrcorner(x_i \circ x_j \circ k_p);h\lrcorner(x_i \circ x_j \circ k_p):=1,$$
$$h\lrcorner\beta_{i,j}:=h\lrcorner(x_i \circ x_j \circ k_p);a\lrcorner\beta_{j,i}:=a\lrcorner(x_j \circ x_i \circ k_p);$$
$$a\lrcorner(x_j \circ x_i \circ k_p):=1/a\lrcorner(x_i \circ x_j \circ k_p)$$
$$h\lrcorner(x_j \circ x_i \circ k_p):=1;h\lrcorner\beta_{j,i}:=h\lrcorner(x_j \circ x_i \circ k_p)\rangle.$$

The operations on attributes of the next rule check the PCM completion of alternatives by p-th criterion. If everything is completed, then the conformity is calculated and the alternative priority vector is filled, otherwise the rule does not apply:

$$s_{11,0,0}=\langle\ \prod_{i=1}^{N}\prod_{j=1}^{N}(x_i \circ x_j \circ k_p)\ \ {}_{d_{11,0,0}}\!\!\rightarrow$$
$$\rightarrow\prod_{i=1}^{N}\prod_{j=1}^{N}(x_i \circ x_j \circ k_p)\cdot\overline{{}_{Q}\lambda\rangle},$$
$${}_{\tau_0}g_{11}=\langle full:=1,d_{11,0,0}:=1,$$
$$\prod_{i=1}^{N}\prod_{j=1}^{N}(\div(a\lrcorner(x_i \circ x_j \circ k_p)\le 0;$$
$$1;full:=0)),\div(full=0;1;d_{11,0,0}:=0)\rangle,$$

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$$_{\tau_1}g_{11,0,0} = \left\langle is \hookleftarrow \overline{\lambda} := \hbar({}_{p,N}\overline{\overline{\beta}});\ \overline{r} \hookleftarrow \overline{\lambda} := \lambda({}_{p,N}\overline{\overline{\beta}}) \right\rangle.$$

The relation $s_{12,0,0}$ is used to generate a sequence of alternatives, if the PCM of alternatives has a valid conformity level. After the substitution on the basis of operations on attributes, the ranks of alternatives are determined:

$$s_{12,0,0} = \langle \prod_{i=1}^{N}\prod_{j=1}^{N}(x_i \circ x_j \circ k_p) \cdot {}_{p,N}\overline{\overline{\beta}} \cdot_Q \overline{\lambda} \cdot \mu \quad {}_{d_{12,0,0}} \rightarrow$$

$$\rightarrow {}_{p,N}\overline{\overline{\beta}} \cdot \prod_{i=1}^{N}({}_{\overline{q}}x_i) \cdot {}_Q\overline{\lambda} \rangle,$$

$$_{\tau_0}g_{12,0,0} = \left\langle \div((is \hookleftarrow \overline{\lambda}) \le 0{,}011;\ 1; d_{12,0,0} := 1) \right\rangle,$$

$$_{\tau_1}g_{12,0,0} = \left\langle \prod_{i=1}^{N}(v \hookleftarrow x_i := r_i \hookleftarrow \overline{\lambda}); d_{12,0,0} := 0 \right\rangle.$$

Implementation of CPS for a single-level AHP is the ranked list of alternatives, the completed PCM and the calculated conformity relation values for the formed matrix.

Originality and practical value

The developed model of constructive process for alternative ranking by modified AHPS can solve the problem with a large number of criteria and alternatives (more than ten), and can also be used under conditions of incomplete information, as part of evaluations is entered by an expert, and the part is calculated based on the input. This method can improve the conformity of expert judgments. CPS-modeling opens wide possibilities for automated hybridization of AHP modifications taking into account the specifics of the tasks.

Conclusions

The developed modeling system for constructive alternatives ranking process consists of three CPS, interacting at different levels of refinement transformations. Disaggregation of process components makes it possible to independently change some models, change their interpretation, which allows applying this approach to solve more specific tasks.

CPS-formalization allows moving to a higher level of abstraction when describing a method for decision making problem solving, which in turn provides an opportunity for the development of programs that implement the hybrid modification of the decision-making methods, in particular the various modifications of AHP.

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МОДЕЛЮВАННЯ МОДИФІКОВАНОГО МЕТОДУ АНАЛІЗУ ІЄРАРХІЙ ЗАСОБАМИ КОНСТРУКТИВНО-ПРОДУКЦІЙНИХ СТРУКТУР

Мета. У дослідженні передбачається: 1) розширити можливості класичного методу аналізу ієрархій (МАІ) для великої кількості альтернатив та критеріїв; 2) побудувати модель конструктивного процесу прийняття рішень із використанням модифікованого методу аналізу ієрархій із сортуванням (МАІС). **Методика.** Для досягнення поставленої мети використовується механізм конструктивно-продукційних структур (КПС). Виконано уточнюючі перетворення узагальнюючої конструктивно-продукційної структури (УКПС). **Результати.** Розроблена модель конструктивного процесу представляє собою взаємодію трьох структур: 1) загальної структури КПС МАІС, яка дозволяє задати альтернативи та критерії, виконуючи декомпозицію ієрархічної структури задачі; 2) КПС групування та сортування, яка розбиває альтернативи (критерії) на групи та реалізує класичний однорівневий МАІ для кожної групи, а також розраховує оцінки парних порівнянь на основі введених даних; 3) КПС однорівневого класичного МАІ, яка дозволяє заповнити матрицю парних порівнянь та розрахувати ранги альтернатив. Всі три структури взаємодіють між собою на різних рівнях уточнюючих перетворень: через узгодження по даним на рівні конкретизації та використання реалізацій. Запропонована модель дозволила перейти на більш абстрактний рівень представлення розв'язку задач прийняття рішень для великої кількості критеріїв та альтернатив. **Наукова новизна**. За результатами роботи пропонується використовувати механізм КПС для формалізації модифікацій МАІ із сортуванням для розв'язку задач прийняття рішень із великою кількістю критеріїв та альтернатив. **Практична значимість.** Формалізація представлення як самого методу аналізу ієрархій, так і його модифікацій дозволяє розширити коло застосування даного методу, впорядкувати описи різних модифікацій МАІ. Таке представлення забезпечує можливість розробки програм для реалізації гібридних модифікацій методу. Використання різних інтерпретацій запропонованих в статті КПС дозволить використати інші підходи при визначенні узгодженості матриць парних порівнянь, розрахунку оцінок та рангів альтернатив і критеріїв.

Ключові слова: моделювання; конструктивно-продукційні структури; конструктивний процес; метод аналізу ієрархій; модифікація

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МОДЕЛИРОВАНИЕ МОДИФИЦИРОВАННОГО МЕТОДА АНАЛИЗА ИЕРАРХИЙ СРЕДСТВАМИ КОНСТРУКТИВНО-ПРОДУКЦИОННЫХ СТРУКТУР

Цель. В исследовании предполагается: 1) расширить возможности классического метода анализа иерархий (МАИ) для большого количества альтернатив и критериев; 2) построить модель конструктивного процесса принятия решений с использованием модифицированного метода анализа иерархий с сортировкой (МАИС). **Методика.** Для достижения поставленной цели используется механизм конструктивно-продукционных структур (КПС). Выполнены уточняющие преобразования обобщенной конструктивно-продукционной структуры. **Результаты.** Разработанная модель конструктивного процесса представляет собой взаимодействие трех структур: 1) общей КПС МАИС, которая позволяет определить альтернативы и критерии, выполняя декомпозицию иерархической структуры задачи; 2) КПС группировки и сортировки, которая разбивает на группы альтернативы и критерии, реализуя для каждой из групп классический одноуровневый МАИ, а также рассчитывая оценки парных сравнений на основании введенных данных; 3) КПС одноуровневого классического МАИ, которая позволяет заполнить матрицу парных сравнений и рассчитать ранги альтернатив. Все три структуры взаимодействуют между собой на разных уровнях уточняющих преобразований: посредством согласования по данным на уровне конкретизации и использования реализаций. Предложенная модель позволила перейти на более абстрактный уровень представления разрешения задач принятия решений для большого количества критериев и альтернатив. **Научная новизна**. По результатам работы предлагается использовать механизм КПС для формализации модификаций МАИ с сортировкой для разрешения задач принятия решений с большим количеством критериев и альтернатив. **Практическая значимость.** Формализация представления как самого метода анализа иерархий, так и его модификаций позволяет расширить круг применения данного метода; унифицировать описания различных модификаций МАИ. Такое представление обеспечивает возможность разработки программ для реализации гибридных модификаций данного метода. Использование разных интерпретаций представленных в статье КПС позволит использовать другие подходы при определении согласованности матриц парных сравнений, расчета оценок и весов альтернатив и критериев.

Ключевые слова: моделирование; конструктивно-продукционные структуры; конструктивный процесс; метод анализа иерархий; модификация

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